Class XII Session 2025-26
Subject - Applied Mathematics
Sample Question Paper - 9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study-based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and one sub-part each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If the points (1, 3) (x, 5) and (2, 7) are collinear, then the value of x is **[1]**

   a) $\frac{3}{4}$ b) $\frac{3}{2}$

   c) 2 d) 1

2. Inferential statistics is a process that involves all of the following except **[1]**

   a) estimating a statistic b) test a hypothesis

   c) estimating a parameter d) analyse relationships

3. The formula $\left[\frac{(1+i)^n-1}{\frac{i}{1+i}}\right]$ is used to calculate [A is amount of each annuity, $i = \frac{r}{100}$, r is rate %, n is time period] **[1]**

   a) sinking value of annuity b) annual value of annuity

   c) present value of annuity due d) future value of annuity due

4. If the constraints in a linear programming problem are changed **[1]**

   a) the objective function has to be modified b) solution is not defined

   c) the problem is to be re-evaluated d) the change in constraints is ignored

[1]

5. If A = $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det (adj (adj A)) is

a) 14^2
b) 14
c) 14^3
d) 14^4

6. Fifteen coupons are numbered 1 to 15. Seven coupons are selected at random, one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is **[1]**

a) none of these
b) $\left(\frac{8}{15}\right)^7$
c) $\left(\frac{3}{5}\right)^7$
d) $\left(\frac{1}{15}\right)^7$

7. There are 50 telephone lines in an exchange. The probability that any one of them will be busy is 0.1. The probability that all the lines are busy is **[1]**

a) $\frac{5^0 e^{-5}}{0!}$
b) 1 - $\frac{5^0 e^{-5}}{0!}$
c) $\frac{5^{50} e^{-5}}{50!}$
d) 1 - $\frac{5^{50} e^{-5}}{50!}$

8. The differential equation x$\frac{dy}{dx}$ - y = x^2, has the general solution: **[1]**

a) 2y + x^2 = 2cx
b) 2y - x^3 = cx
c) y + x^2 = 2cx
d) y - x^3 = 2cx

9. If a man rows 32 km downstream and 14 km upstream in 6 hours each, then the speed of the stream is **[1]**

a) 2.5 km/h
b) 2 km/h
c) 2.25km/h
d) 1.5 km/h

10. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the value of x is **[1]**

a) 3
b) 6
c) ± 3
d) ± 6

11. (8 $\times$ 14) in 12 hours clock is **[1]**

a) 8 O'clock
b) 2 O'clock
c) 4 O'clock
d) 6 O'clock

12. If $\frac{|x+1|}{x+1} > 0$, x $\in$ R, then **[1]**

a) x $\in$ $(-\infty$, -1]
b) x $\in$ [-1, ∞)
c) x $\in$ $(-\infty$, -1)
d) x $\in$ (-1, ∞)

13. A man rows 40 km upstream in 8 hours and a distance of 36 km downstream in 6 hours. The speed of the stream is **[1]**

a) $\frac{1}{2}$ km/hr
b) $5\frac{1}{2}$km/hr
c) 11 km/hr
d) $\frac{1}{2}$km/hr

14. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is z = 4x + 3y. **[1]**

Compare the quantity in Column A and Column B

Column A	Column B
Maximum of z	325

a) The quantity in Column B is greater
b) The quantity in column A is greater
c) The two quantities are equal
d) The relationship can not be determined on the basis of information supplied

15. The point in the half-plane 2x + 3y −12 $\geq$ 0 is: **[1]**
a) (7, 8)
b) (-7, -8)
c) (7, -8)
d) (-7, 8)

16. If we reject the null hypothesis, we might be making **[1]**
a) A correct decision
b) A wrong decision
c) Type-II error
d) Type-I error

17. If $\int_0^{40} \frac{dx}{2x+1} = \log k$, then the value of k is **[1]**
a) 9
b) 3
c) 5
d) $\frac{9}{2}$

18. The straight line trend is represented by the equation: **[1]**
a) y_c = na - $b\Sigma x$
b) y_c = na + $b\Sigma x$
c) y_c = a + bx
d) y_c = a - bx

19. **Assertion (A):** Scalar matrix A = $[a_{ij}] = \begin{cases} k, & i = j \\ 0, & i \neq j \end{cases}$, where k is a scalar, is an identity matrix when k = 1. **[1]**
Reason (R): Every identity matrix is not a scalar matrix.
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The tangent at x = 1 to the curve y = x^3 - x^2 - x + 2 again meets the curve at x = -2. **[1]**
Reason (R): When a equation of a tangent solved with the curve, repeated roots are obtained at point of tangency.
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. A dining table costing ₹ 36000 has a useful life of 15 years. If annual depreciation is ₹ 2000, find its scrap value using the linear method. **[2]**

OR

A banker credits the fixed deposit account of a depositor on a continuous basis. As a result, the effective rate of interest earned by a depositor is 9.43%. Find out the rate of interest that is allowed by the banker. What is the effective rate of interest if it is compounded on quarterly basis?

22. From the following data compute 4-yearly moving averages and determine the trend values. Also, find the short- **[2]**

term fluctuations.

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Value:	50	36.5	43.0	44.5	38.9	38.1	32.6	41.7	41.1	33.8

23. By using property of definite integrals, evaluate $\int_0^3 x^2\sqrt{3-x}dx$ **[2]**

24. Find A (adj A) for the matrix A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$. **[2]**

OR

Let f(x) = x^2 - 5x + 6. Find f(A), if A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

25. Two equal glasses filled with mixtures of alcohol and water in the proportions 2 : 1 and 1 : 1 respectively were emptied into a third glass. What is the proportion of alcohol and water in the third glass? **[2]**

Section C

26. Find the effective rate that is equivalent to a nominal rate of 8% compounded: **[3]**

i. semi-annually
ii. quarterly
iii. continuosly

27. Write the order of the differential equation of the family of circles of radius r. **[3]**

OR

Solve the differential equation: x log x $\frac{dy}{dx}$ + y = $\frac{2}{x}$ log x

28. The marginal revenue function of a firm is given by MR = $\frac{ab}{(x-b)^2} - c$. Prove that the total revenue function and the demand function are given by R = $\frac{ab}{b-x} - cx - a$ and p = $\frac{a}{b-x} - c$ respectively. **[3]**

29. From the following time series obtain trent value by 3 yearly moving averages. **[3]**

Year	**Sales (in ₹ 000)**	**Year**	**Sales (in ₹ 000)**
2008	8	2014	16
2009	12	2015	17
2010	10	2016	14
2011	13	2017	17
2012	15		
2013	12		

30. Consider the following hypothesis test: **[3]**

$H_0 : p \geq 0.75$

$H_a : p < 0.75$

A sample of 300 provided a sample proportion of 0.68.

i. Compute the value of the test statistic.
ii. What is the p-value?
iii. At α = 0.05, what is your conclusion?
iv. What is the rejection rule using critical value? What is your conclusion?

31. Students of a class were given a mechanical aptitude test. Their marks were found to be normally distributed w^{th} **[3]**
mean 60 and standard deviation 5. What percent of students scored

i. more than 60 marks?
ii. less than 56 marks?
iii. between 45 and 65 marks?

OR

Two cards are drawn without replacement from a well-shuffled deck of 52 cards. Determine the probability distribution of the number of face cards (i.e. Jack, Queen, King, and Ace).

Section D

32. Find the maximum value of 7x + 6y subject to the constraints: **[5]**

$x + y \geq 2$

$2x + 3y \leq 6$

$x \geq 0$ and $y \geq 0$.

OR

A farmer has a 100 acre farm. He can sell the tomatoes, lettuce, or radishes he can raise. The price he can obtain is ₹ 1 per kilogram for tomatoes, ₹ 0.75 a head for lettuce and ₹ 2 per kilogram for radishes. The average yield per acre is 2000 kgs for radishes, 3000 heads of lettuce and 1000 kilograms of radishes. Fertilizer is available at ₹ 0.50 per kg and the amount required per acre is 100 kgs each for tomatoes and lettuce and 50 kilograms for radishes. Labour required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man-days of labour are available at ₹ 20 per man-day. Formulate this problem as an LPP to maximize the farmer's total profit.

33. The probability that Rohit will hit a shooting target is $\frac{2}{3}$. While preparing for an international shooting competition. Rohit aims to achieve the probability of hitting the target atleast once to be 0.99. What is the minimum number of chances must he shoot to attain this probability? **[5]**

OR

Two positive numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and the variance of the distribution.

34. A milkman has two cans. First containing 75%milk and rest water, whereas second containing 50% milk and rest water. How much mixture should he mix from each can so as to get 20 litres of mixture, such that ratio of milk and water is 5 : 3 respectively? **[5]**

35. A machine costing ₹ 200000 has effective life 7 years and its scrap value is ₹ 30000. What amount should the company put into a sinking fund earning 5% per annum, so that it can replace the machine after its useful life? Assume that a new machine will cost ₹ 300000 after 7 years. [Given log (1.05) = 0.0212 and antilog (0.1484) = 1.407] **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

The demand function for a certain product is represented by the equation: $p = ax^2 + bx + c$, where x is the number of units demanded and p is the price per unit.

i. Find the revenue function R(x). (1)
ii. Find the marginal revenue MR. (1)
iii. Find the values of x, for which marginal revenue increases. (2)

OR

Find the values of x for which marginal revenue decreases. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

In year 2000, Mr. Talwar took a home loan of ₹ 30,00,000 from State Bank of India at 7.5 % p.a. compounded monthly for 20 years.

i. Find the equated monthly instalment paid by Mr. Talwar. (1)
ii. Find the interest paid by Mr. Talwar in 150th payment. (1)
iii. Find the principal paid by Mr. Talwar in 150th payment. (2)

OR

Find the total interest paid by Mr. Talwar. (2)

[Use $(1.00625)^{240} = 4.4608$, $(1.00625)^{91} = 1.7629$, $(1.00625)^{48} = 1.1187$]

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Amit, Biraj and Chirag were given the task of creating a square matrix of order 2.

Below are the matrices created by them. A, B, C are the matrices created by Amit, Biraj and Chirag respectively.

$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

If a = 4 and b = −2, based on the above information answer the following:

i. What will be the sum of the matrices A, B and C? (1)
ii. What is the value of $(bA)^T$? (1)
iii. What is the value of AC − BC? (2)

OR

What is the value of (a + b)B? (2)

Solution

Section A

1.
(b) $\frac{3}{2}$
Explanation:
$\begin{vmatrix} 1 & 3 & 1 \\ x & 5 & 1 \\ 2 & 7 & 1 \end{vmatrix}$ = 0 $\Rightarrow$ 1(5 - 7)-3(x - 2) + 1(7x - 10) = 0
$\Rightarrow$ -2 - 3x + 6 + 7x - 10 = 0
$\Rightarrow$ 4x - 6 = 0
$\Rightarrow$ x = $\frac{3}{2}$

2. **(a)** estimating a statistic
Explanation:
estimating a statistic

3.
(d) future value of annuity due
Explanation:
future value of annuity due

4.
(c) the problem is to be re-evaluated
Explanation:
The constraints of a linear programming problem are changed.
Now, as per the definition of the Linear Programming Problem,
A Linear programming problem is a linear function (also known an objective function) subjected to certain constraints for which we need to find an optimal solution (i.e. either a maximum/minimum value) depending on the requirement of the problem.
Here, the LPP is solved using the constraints, so, if the constraints are changed, the problem is to has to be re-calculated with the new constraints provided.

5.
(d) 14^4
Explanation:
A = $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$
|A| = 14 det(adjA) = det$(A)^{3-1}$ = det$(A)^2$. Here the operation is done two times. So,
det (adj(adj A)) = $|A|^{(n-1)^2}$
det (adj(adj A)) = $14^{(3-1)^2}$ = 14^4

6. **(a)** none of these
Explanation:
The sample space = 15^7 for selecting seven coupons from 15 coupons.
The maximum number on the selected coupon is 9 can be made by 9^7 Ways.

A number selected on second card is less than 9 can be made by 8^7 ways.

Required probability $= \frac{9^7 - 8^7}{15^7}$

7.

(c) $\frac{5^{50} e^{-5}}{50!}$

Explanation:

Given n = 50, P = 0.1, so λ = np = 50 $\times$ 0.1 = 5

So, P(X = 50) $= \frac{5^{50} . e^{-5}}{50!}$

8.

(b) 2y - x^3 = cx

Explanation:

We have,

$x\frac{dy}{dx}$ - y = x^2

$\Rightarrow \frac{dy}{dx} - \frac{y}{x}$ = x

Comparing with $\frac{dy}{dx}$ - Py = Q

$\Rightarrow P = \frac{-1}{x}$, Q = x

I.F. = $e^{\int P dx} = e^{\int \frac{-1}{x} dx} = e^{-\log x} = \frac{1}{x}$

Multiplying $\frac{1}{x}$ on both sides,

$\frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2}$ = 1

$\frac{d}{dx} \frac{y}{x}$ = 1

$\int \frac{d}{dx} \frac{y}{x} = \int x dx$

$\frac{y}{x} = \frac{x^2}{2}$ + c

2y = x^3 + cx

2y - x^3 - cx = 0

9.

(d) 1.5 km/h

Explanation:

Downstream speed u $= \frac{32}{6}$ km/h $= \frac{16}{3}$ km/h

and upstream speed v $= \frac{14}{6}$ km/h $= \frac{7}{3}$ km/h

So, speed of the stream $= \frac{u-v}{2} = \frac{\frac{16}{3} - \frac{7}{3}}{2}$ km/h = 1.5 km/h

10.

(d) ± 6

Explanation:

as $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$2x^2 - 40$ = 18 + 14

x = ± 6

11.

(c) 4 O'clock

Explanation:

(8 $\times$ 14) (mod 12) = 112 (mod 12) = 4 i.e. 4 O' clock

12.

(d) x $\in$ (-1, ∞)

Explanation:

$\frac{|x+1|}{x+1} > 0 \Rightarrow x + 1 > 0 \Rightarrow x > -1$

$\therefore x \in (-1, \infty)$

13. **(a)** $\frac{1}{2}$ km/hr

Explanation:

Let the speed of man and stream be x and y km/hr

downstream speed = (x - y) km/hr

upstream speed = (x + y) km/hr

$t_{\text{upstream}} = \frac{40}{x-y}$

8(x - y) = 40

x - y = 5 ...(i)

$t_{\text{downstream}} = \frac{36}{x+y}$

6(x + y) = 36

x + y = 6 ...(ii)

(i) - (ii)

-2y = -1

y = $\frac{1}{2}$

Hence, speed of stream = $\frac{1}{2}$ km/hr

14. **(a)** The quantity in Column B is greater

Explanation:

Corner Points	Corresponding Value of Z = 4x + 3y
(0, 0)	0
(0, 40)	120
(20, 40)	200
(60, 20)	300 **(Maximum)**
(60, 0)	240

So, maximum value of Z = 300 < 325.

Therefore, the quantity in column B is greater.

Which is the required solution.

15. **(a)** (7, 8)

Explanation:

(7, 8)

16.

(d) Type-I error

Explanation:

Type-I error

17. **(a)** 9

Explanation:

$$\int_0^{40} \frac{dx}{2x+1} = \log k \Rightarrow \left[\frac{1}{2}\log|2x+1|\right]_0^{40} = \log k \Rightarrow \frac{1}{2}\log 81 = \log k$$

$$\Rightarrow \log\sqrt{81} = \log k \Rightarrow k = 9$$

18.

(c) $y_c = a + bx$

Explanation:

$y_c = a + bx$

19.

(c) A is true but R is false.

Explanation:

A scalar matrix A = $[a_{ij}] = \begin{cases} k; & i = j \\ 0; & i \neq j \end{cases}$ is an identity matrix when k = 1.

But every identity matrix is clearly a scalar matrix.

20.

(d) A is false but R is true.

Explanation:

When x = 1, then y = $(1)^3 - (1)^3 - 1 + 2 = 1$

$\therefore \frac{dy}{dx} = 3x^2 - 2x - 1 \Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = 0$

$\therefore$ Equation of tangent at point (1, 1) is

y - 1 = 0(x - 1) $\Rightarrow$ y = 1

Solving with the curve, $x^3 - x^2 - x + 2 = 1$

$\Rightarrow x^3 - x^2 - x + 1 = 0$

$\Rightarrow (x - 1)(x^2 - 1) = 0 \Rightarrow = 1, 1, -1$ [here, 1 is repeated root]

$\therefore$ Tangent meets the curve again at x = -1

$\therefore$ Assertion is false, Reason is true.

Section B

21. Given original cost of dining table = ₹36000

Useful life = 15 years

Annual depreciation = ₹2000

Let the scrap value of the dining table be ₹S, then using

annual depreciation = $\frac{\text{original cost - scrap value}}{\text{usetul life}}$, we get

$2000 = \frac{36000-S}{15}$

$\Rightarrow$ 30000 = 36000 - S

$\Rightarrow$ S = 6000

Hence, the scrap value of the dining table is ₹6000.

OR

Let the rate of interest allowed by the banker be r. It is given that $r_e = \frac{9.43}{100} = 0.0943$

$\therefore r = 2.3025 \log (1 + r_e)$

$\Rightarrow r = 2.3025 \log (1.0943) = 2.3025 \times 0.0391 = 0.0900$

Thus, the rate of interest allowed by the banker is 9% compounded continuously

If the interest is compounded quarterly, then

r = 0.09, m = 4

$\therefore r_e = (1 + \frac{r}{m})^m - 1$

$\Rightarrow r_e = (1 + \frac{0.09}{4})^4 - 1 = (1.0225)^4 - 1 = 1.0930 - 1 = 0.0930$

Thus, the effective rate of interest is 9.3%.

22. Computation of4-yearly moving averages.

Year	Value	4-yearly moving totals	4-yearly centered moving averages	Short term fluctuations $Y - Y_c$
2006	50.0	-	-	
2007	36.5	-	-	
		$\leftarrow$ 174.0	43.5	
			$\leftarrow$ 42.1125	0.8875

2008	43.0			
		$\leftarrow$ 162.9	40.725	
			$\leftarrow$ 40.925	3.575
2009	44.5			
		$\leftarrow$ 164.5	41.125	
			$\leftarrow$ 39.825	-0.925
2010	38.9			
		$\leftarrow$ 154.1	38.525	
			$\leftarrow$ 38.175	-0.075
2011	38.1			
		$\leftarrow$ 151.3	37.825	
			$\leftarrow$ 38.1	-5.5
2012	32.6			
		$\leftarrow$ 153.5	38.375	
			$\leftarrow$ 37.8375	3.8625
2013	41.7	$\leftarrow$ 149.2	37.3	
2014	41.1	-	-	
2015	33.8	-	-	

23. $\int_0^3 x^2\sqrt{3-x}dx = \int_0^3 (3-x)^2(3-(3-x))^{\frac{1}{2}}dx$ (by property P_4)

$= \int_0^3 (9-6x+x^2)\, x^{\frac{1}{2}}dx = \int_0^3 \left(9x^{\frac{1}{2}} - 6x^{\frac{3}{2}} + x^{\frac{5}{2}}\right) dx$

$= \left[9 \times \frac{2}{3}x^{\frac{3}{2}} - 6 \times \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{7}x^{\frac{7}{2}}\right]_0^3$

$= 6 \cdot 3\sqrt{3} - \frac{12}{5} \cdot 9\sqrt{3} + \frac{2}{7} \cdot 27\sqrt{3} = \frac{144}{35}\sqrt{3}.$

24. Here, A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$

Cofactors of A are:

C_{11} = 9, C_{21} = 19, C_{31} = -4, C_{12} = 4, C_{22} = 14, C_{32} = 1, C_{13} = 8, C_{23} = 3, C_{33} = 2

$\therefore$ adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

= $\begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}^T$

$\therefore$ adj A = $\begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix}$

Now, A adj A = $\begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix} \begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix}$

= $\begin{bmatrix} 25 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 25 \end{bmatrix}$

$= 25 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= 25\ I_3$

OR

First, we note that by f(A) we mean the matrix polynomial $A^2 - 5A + 6I_3$. That is, to obtain f(A), x is replaced by A and the constant term is multiplied by the identity matrix of order same as that of A.

Now, $A^2 = AA = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$

$-5A = \begin{bmatrix} (-5)\times 2 & (-5)\times 0 & (-5)\times 1 \\ (-5)\times 2 & (-5)\times 1 & (-5)\times 3 \\ (-5)\times 1 & (-5)\times(-1) & (-5)\times 0 \end{bmatrix} = \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}$

and, $6I_3 = 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$\therefore f(A) = A^2 - 5A + 6I_3 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$\Rightarrow f(A) = A^2 - 5A + 6I_3 = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$

25. Two ingredients A and B are in the ratio $a_1 : b_1$ and $a_2 : b_2$ in two mixtures M_1 and M_2 respectively. If q_1 units of M_1 and q_2 units of M_2 are mixed to form a resultant mixture M, then the ingredients A and B in the resultant mixture are in the ratio

$\frac{q_A}{q_B} = \frac{\left(\frac{a_1}{a_1+b_1}\right)q_1 + \left(\frac{a_2}{a_2+b_2}\right)q_2}{\left(\frac{b_1}{a_1+b_1}\right)q_1 + \left(\frac{b_2}{a_2+b_2}\right)q_2}$

Hence, $a_1 = 2$, $b_1 = 1$, $a_2 = 1$, $b_2 = 1$ and $q_1 = q_2$ (glasses are equal)

$\therefore \frac{\text{Alcohol}}{\text{Water}} = \frac{\left(\frac{2}{2+1}\right)q_1 + \left(\frac{1}{1+1}\right)q_1}{\left(\frac{1}{2+1}\right)q_1 + \left(\frac{1}{1+1}\right)q_1} = \frac{\frac{2}{3}+\frac{1}{2}}{\frac{1}{3}+\frac{1}{2}} = \frac{7}{5}$

Hence, in the third glass alcohol and water are in the ratio 7 : 5.

Section C

26. i. We have, $r = \frac{8}{100} = 0.08$, $m = 2$

$\therefore r_e = (1 + \frac{r}{m})^m - 1$

$\Rightarrow r_e = (1 + \frac{0.08}{2})^2 - 1 = (1.04)^2 - 1 = 1.0816 - 1 = 0.0816$

Hence, the effective rate is 8.16% which means that the rate 8.16% compounded annually yields the same interest as the nominal rate 8% compounded semi-annually.

ii. We have, $r = \frac{8}{100} = 0.08$, $m = 4$

$\therefore r_e = (1 + \frac{r}{m})^m - 1$

$\Rightarrow r_e = (1 + \frac{0.08}{4})^4 - 1 = (1.02)^4 - 1 = 1.08243216 - 1 = 0.08243216$

Hence, the effective rate of interest is 8.24% which means that the rate 8.24% compounded annually yields the same interest as the nominal rate 8% compounded quarterly.

iii. We have, $r = \frac{8}{100} = 0.08$

$\therefore r_e = e^r - 1 \Rightarrow r_e = e^{0.08} - 1 = 1.0833 - 1 = 0.0833$

Hence, the effective rate is 8.33% . This means that the rate 8.33% compounded annually yields the same interest as the nominal rate 8% compounded continuously.

27. We know that, family of circle with given radius r and centre (h, k) is given by,

$(x - h)^2 + (y - k)^2 = r^2$ ...(i)

Differentiating it with respect to x,

$2(x - h) + 2(y - k) \frac{dy}{dx} = 0$

$(x - h) + (y - k) \frac{dy}{dx} = 0$ ...(ii)

Again, differentiating it with respect to x,

$1 + (y - k) \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

$(y - k) = \frac{-\left(\frac{dy}{dx}\right)^2 - 1}{\frac{d^2y}{dx^2}}$ ...(iii)

put the value of (y - k) from equation (iii) in equation (ii)

$(x - h) + \left[\frac{-\left(\frac{dy}{dx}\right)^2 - 1}{\frac{d^2y}{dx^2}}\right] \frac{dy}{dx} = 0$

$(x - h) - \left[\frac{\left(\frac{dy}{dx}\right)^2 + 1}{\frac{d^2y}{dx^2}}\right] \frac{dy}{dx} = 0$

$(x - h) = \left[\frac{\left(\frac{dy}{dx}\right)^2 + 1}{\frac{d^2y}{dx^2}}\right] \frac{dy}{dx}$

Put the value of (x - h) and (y - k) is equation (i)

$\left\{\left[\frac{\left(\frac{dy}{dx}\right)^2 + 1}{\frac{d^2y}{dx^2}}\right]\left(\frac{dy}{dx}\right)\right\}^2 + \left\{-\left[\frac{\left(\frac{dy}{dx}\right)^2 + 1}{\frac{d^2y}{dx^2}}\right]\right\} = r^2$

$\left[\left(\frac{dy}{dx}\right)^2 + 1\right]^2 + \left[\left(\frac{dy}{dx}\right)^2 + 1\right] = r^2 \frac{d^2y}{dx^2}$

$r^2 \frac{d^2y}{dx^2} - \left[\left(\frac{dy}{dx}\right)^2 + 1\right]^3 = 0$

The highest order derivative appearing in this differential equation is $\frac{d^2y}{dx^2}$, therefore, we have,

The order of the differential equation is 2.

OR

The given differential equation is

$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x^2}$ ...(i)

This is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x \log x}$ and $Q = \frac{2}{x^2}$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \frac{1}{x \log x} dx} = e^{\int \frac{1}{t} dt}$, where t = log x

$\Rightarrow$ I.F. $= e^{\log t} = t = \log x$

Multiplying both sides of (i) by I.F. = log x, we get

$\log x \frac{dy}{dx} + \frac{1}{x} y = \frac{2}{x^2} \log x$

Integrating both sides with respect to x, we get

$y \log x = \int \frac{2}{x^2} \log x \, dx + C$ [Using: $y(\text{I.F.}) = \int Q (\text{I.F.}) dx + c$]

$\Rightarrow y \log x = 2 \int \underset{I}{\log x}\ \underset{II}{x^{-2}} dx + C$

$\Rightarrow y \log x = 2 \left\{\log x \left(\frac{x^{-1}}{-1}\right) - \int \frac{1}{x}\left(\frac{x^{-1}}{-1}\right) dx\right\} + C$

$\Rightarrow y \log x = 2\left\{-\frac{\log x}{x} + \int x^{-2} dx\right\} + C$

$\Rightarrow y \log x = 2\left\{-\frac{\log x}{x} - \frac{1}{x}\right\} + C$

$\Rightarrow y \log x = -\frac{2}{x}(1 + \log x) + C$, which gives the required solution.

28. We have,

MR = $\frac{ab}{(x-b)^2} - c \Rightarrow \frac{dR}{dx} = \frac{ab}{(x-b)^2} - c$

Integrating both sides with respect to x, we get

R = $\int \left\{ \frac{ab}{(x-b)^2} - c \right\}$ dx + k

$\Rightarrow$ R = $-\frac{ab}{x-b}$ - cx + k ... (i)

When x = 0, we have R = 0. Putting x = 0 and R = 0 in (i), we get

0 = a + k $\Rightarrow$ k = -a

Putting k = -a in (i), we get

R = $-\frac{ab}{x-b}$ - cx - a

$\Rightarrow$ R = $\frac{ab}{b-x}$ - cx - a, which is the total revenue function.

Let p be the price per unit when x units of the product are sold. Then,

R = px $\Rightarrow$ p = $\frac{R}{x}$ $\Rightarrow$ p = $\frac{ab}{x(b-x)} - c - \frac{a}{x}$

$\Rightarrow$ p = $\frac{a}{b-x}$ - c, which is the required demand function.

29. Calculating of trend values by three yearly moving average method.

Year	**Sales (Thousand ₹)**	**Three-yearly Moving Totals**	**Three-yearly Moving Average (Trend value)**
2008	8		
2009	12	(8 + 12 + 10) = 30	10.00
2010	10	(12 + 10 + 13) = 35	11.67
2011	13	(10 + 13 + 15) = 38	12.67
2012	15	(13 + 15 + 12) = 40	13.33
2013	12	(15 + 12 + 16) = 43	14.33
2014	16	(12 + 16 + 17) = 45	15.00
2015	17	(16 + 17 + 14) = 47	15.67
2016	14	(17 + 14 + 17) = 48	16.00
2017	17		

30. Given p_0 = 0.75, n = 300, $\bar{p}$ = 0.68

i. Z = $\frac{\bar{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.68-0.75}{\sqrt{\frac{0.75\times0.25}{300}}}$

= $\frac{-0.07\times10}{\sqrt{0.25\times0.25}} = \frac{-0.07}{0.25}$ = -2.8

ii. $\because$ Z = -2.8 < 0

So, p-value of -2.8 = area under the standard normal curve to the left of Z

= 0.0026

$\therefore$ p-value = 0.0026

iii. Given α = 0.05

$\because$ p-value < 0.05

So, reject H_0.

iv. Rejection rule using critical value

Reject H_0 if Z $\leq$ - Z_α

Here, α = 0.05. So Z_α = $Z_{0.05}$ = 1.645

$\Rightarrow -Z_\alpha$ = -1.645

$\because$ -2.8 < -1.645

$\Rightarrow$ Z < -Z_α

So, reject H_0

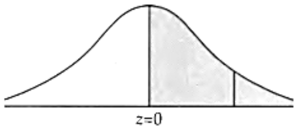
31. Let X denote the marks scored by the students. Then X is normally distributed with mean μ = 60 and standard deviation σ = 5.

Let Z be the standard normal variate. Then,

Z = $\frac{X-\mu}{\sigma}$ $\Rightarrow$ Z = $\frac{X-60}{5}$

i. When X = 60, we obtain: Z = $\frac{60-60}{5}$ = 0

$\therefore$ P (X > 60) = P (Z > 0) = 0.5



Thus, 50% of the students scored more than 60 marks.

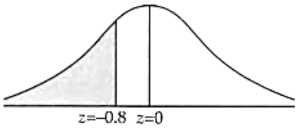
ii. When X = 56, we obtain: Z = $\frac{56-60}{5}$ = -0.8

$\therefore$ P(X < 56)

= P(Z < -0.8)

= P(Z > 0.8)

= 0.5 - P(0 $\leq$ Z < 0.8) = 0.5 - 0.2881 = 0.2119



Thus, 21.19 % of the students scored less than 56 marks.

iii. When X = 45, we obtain: Z = $\frac{45-60}{5}$ = -3

When X = 65, we obtain: Z = $\frac{65-60}{5}$ = 1

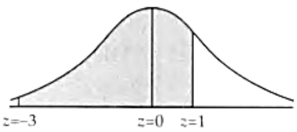
$\therefore$ P(45 < X < 65)

= P(-3 < Z < 1)

= P(-3 < Z $\leq$ 0) + P(0 $\leq$ Z < 1)

= P(0 $\leq$ Z < 3) + P(0 $\leq$ Z < 1)

= 0.4986 + 0.3413 = 0.8399



Thus, 83.99 % of the students scored between 45 and 65 marks.

OR

Let X be a random variable denoting the number of face cards in two draws. Then, X can take values 0, 1, 2.

Let F_i denote the event of getting a face card in i^{th} draw. Then, we have,

P (X = 0) = Probability of getting no face card

$\Rightarrow$ P(X = 0) =$P(\overline{F_1} \cap \overline{F_2})$

$\Rightarrow$ P(X = 0) = $P(E_1)\, P(F_2/F_1)$ [By multiplication theorem]

$\Rightarrow$ P(X = 0)= $\frac{36}{52} \times \frac{35}{51} = \frac{105}{221}$

P(X = 1) = Probability of getting one face card and one other card

$\Rightarrow$ P(X = 1) = $P\left(\left(F_1 \cap \overline{F_2}\right) \cup \left(\overline{F_1} \cap F_2\right)\right)$

$\Rightarrow$ P(X = 1) = $P\left(F_1 \cap \overline{F_2}\right) + P\left(\overline{F_1} \cap F_2\right)$ [By addition theorem]

$\Rightarrow$ P(X =1) = $P(F_1)\, P(\bar{F}_2/F_1) + P(\bar{F}_1)\, P(F_2/\bar{F}_1) = \frac{16}{52} \times \frac{36}{51} + \frac{36}{52} \times \frac{16}{51} = \frac{96}{221}$

P(X = 2) = Probability of getting both face cards

$\Rightarrow$ P(X = 2) = $P(F_1 \cap F_2)$

$\Rightarrow$ P(X = 2) = P (F_1) P (F_2/F_1) = $\frac{16}{52} \times \frac{15}{51} = \frac{20}{221}$

Therefore, the probability distribution is given by,

X	0	1	2
P(X)	$\frac{105}{221}$	$\frac{96}{221}$	$\frac{20}{221}$

Section D

32. x + y $\geqslant$ 2

x + y = 2

x	0	2
y	2	0

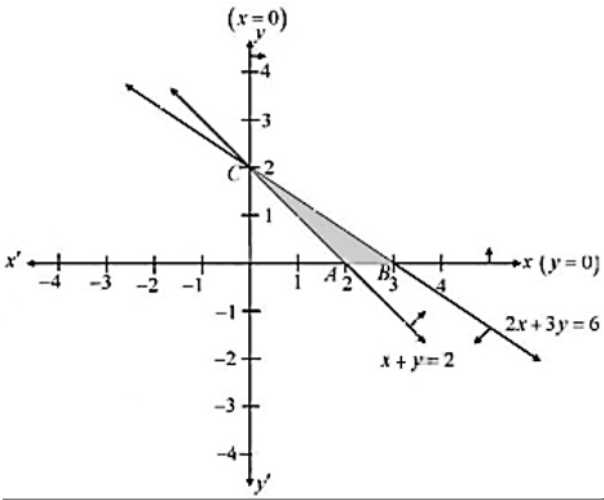
$0 \geqslant 2$ (F)

$2x + 3y \leqslant 6$

$2x + 3y = 6$

x	0	3
y	2	0

$0 \leqslant 6$ (T)



Corner Points	**$Z_{max} = 7x + 6y$**
A (0, 2)	Z = 7(0) + 6(2) = 12
B (2, 0)	Z = 7(2) + 6(0) = 14
C (3, 0)	Z = 7(3) + 6(0) = 21 $\longrightarrow$ Maximum

OR

Given information can be tabulated below

Product	**Yield**	**Cultivation**	**Price**	**Fertilizers**
Tomatoes	2000 kg	5 days	1	100 kg
Lettuce	3000 kg	6 days	0.75	100 kg
Radishes	1000 kg	5 days	2	50 kg

Average = 200 kg/per acre

Total land = 100 Acre

Cost of fertilizers = ₹ 0.50 per kg

A total of 400 days of cultivation labour with ₹ 20 per day.

Let required quantity of field for tomatoes, lettuce and radishes be x, y and z acre respectively.

Given, costs of cultivation and harvesting of tomatoes, lettuce and radishes are 5×20 = ₹100, 6×20 = ₹120, 5×20 = ₹100 respectively per acre. Cost of fertilizers for tomatoes, lettuce and radishes 100×0.50 = ₹ 50, 100×0.50 = ₹50 and 50×0.50 = ₹ 25 respectively per acre.

So, total costs of production of tomatoes, lettuce and radishes are (₹100 + 50) x = ₹ 150 x, (₹ 120 + 50) y = ₹170 y and radishes are (₹100 +25) z = ₹ 125 z respectively total selling price of

tomatoes, lettuce and radishes, according to yield are (₹2000 $\times$ 1) x = ₹2000 x, (₹ 3000 $\times$ 0.75) y = ₹2250 y and (₹1000 $\times$ 2) z = ₹ 2000 z respectively.

Let U be the total profit,

So,

U = (2000x -150x) + (2250y - 170y) + (2000z - 125z)

U = 1850x + 2080y + 1875z

Given, farmer has 100 - acre farm

So, x + y + z $\leq$ 100 (First constraint)

Number of cultivation and harvesting days are 400 So, 5 x + 6y + 5z $\leq$ 400

Hence, mathematical formulation of LPP is,

Find x, y, z which

maximize U = 1850x + 2080y + 1875z

Subject to constraint,

x + y + z $\leq$ 100

5x + 6y + 5z $\leq$ 400

x, y, z $\geq$ 0 [Since farm used for cultivation cannot be less than zero.]

33. Given probability of hitting a shooting target = $p = \frac{2}{3}$.

So, q = 1 - p = $1 - \frac{2}{3} = \frac{1}{3}$.

Let the number of trials be n.

The probability of hitting target atleast once = P(X $\geq$ 1) = 1 - P(0)

$= 1 - {}^nC_0 q^n = 1 - \left(\frac{1}{3}\right)^n$

According to given,

$1 - \left(\frac{1}{3}\right)^n > 0.99 \Rightarrow 1 - \frac{1}{3^n} > \frac{99}{100}$

$\Rightarrow 1 - \frac{99}{100} > \frac{1}{3^n} \Rightarrow \frac{1}{100} > \frac{1}{3^n}$

$\Rightarrow 100 < 3^n$

$\Rightarrow 3^n > 100$, which is satisfied if n is atleast 5.

Hence, Rohit must shoot the target at 5 times.

OR

The number of ways of selecting two numbers from the first five positive integers $= {}^5C_2 = 10$.

So, the sample space S of the random experiment has 10 equally likely outcomes.

The outcomes are:

1, 2; 1, 3; 1, 4; 1, 5; 2, 3; 2, 4; 2, 5; 3, 4; 3, 5; 4, 5.

As the random variable X denote the larger of the two numbers. X can take values 2, 3, 4, 5. Note that in a sample space S, we have

Larger than any number	**Number of outcomes**
2	1
3	2
4	3
5	4

$P(X=2) = \frac{1}{10}, P(X=3) = \frac{2}{10},\ P(X=4) = \frac{3}{10}, P(X=5) = \frac{4}{10}$

$\therefore$ Probability distribution of X is

X	2	3	4	5
P(X)	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

Mean $= \Sigma p_i x_i = \frac{1}{10} \times 2 + \frac{2}{10} \times 3 + \frac{3}{10} \times 4 + \frac{4}{10} \times 5 = 4$

Variance $= \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2$

$= \frac{1}{10}$ $(1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + 4 \times 5^2) - (4)^2$

$= \frac{170}{10}$ - 16 = 17 - 16 = 1.

34. Quantity. of milk in first mixture = $\frac{75}{100} = \frac{3}{4}$ part,

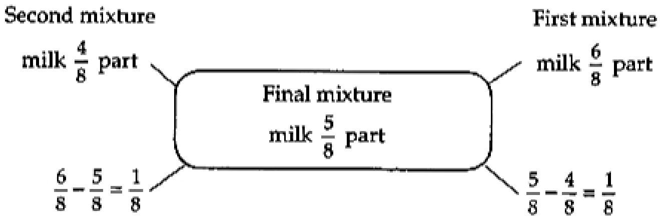
quantity of milk in second mixture = $\frac{50}{100} = \frac{1}{2}$

and quantity of milk in final mixture = $\frac{5}{8}$ part

LCM of 4, 2 and 8 = 8

$\therefore$ Quantity of milk in first mixture = $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$ part

Quantity of milk in second mixture = $\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$ part



$\therefore \frac{\text{Quantity of second mixture}}{\text{Quantity of first mixture}} = \frac{\frac{1}{8}}{\frac{1}{8}} = \frac{1}{1}$ i.e. 1: 1.

Hence, equal quantity of mixture from each can i.e. 10 litres from first can and 10 litres from second can should be mixed.

35. Cost of new machine = ₹ 300000 Scrap value of old machine = ₹ 30,000 Hence, the money required for new machine after 7 years = ₹ 300000 - ₹ 30,000 = ₹ 2,70,000

So, we have A = ₹ 270000 i = $\frac{r}{100} = \frac{5}{100}$ = 0.05, n = 7

Using formula, A = P$\left[\frac{(1+i)^n-1}{i}\right]$ we get

270000 = P$\left[\frac{(1+0.05)^7-1}{0.05}\right]$

$\Rightarrow$ P = $\frac{270000 \times 0.05}{(1.05)^7-1}$

Now, let $(1.05)^7$ = x

Taking log both sides, we get

71og(1.05) = log x

$\Rightarrow$ log x = 7 x 0.0212 = 0.1484

$\Rightarrow$ x = antilog (0.1484) = 1.407

So, (1.05)7 = 1.407

Thus, P = $\frac{270000 \times 0.05}{1.407-1}$

$\Rightarrow$ P=$\frac{13500}{0.407}$

$\Rightarrow$ P = ₹ 33169.53

Hence, the company should deposit 33169.53 at the end of each year for 7 years.

Section E

36. i. Given p = $ax^2 + bx + c$

$\Rightarrow$ R(x) = px $\Rightarrow$ R(x) = $ax^3 + bx^2 + cx$.

ii. Marginal revenue MR = $\frac{d}{dx}$R(x) = $\frac{d}{dx}$ $(ax^3 + bx^2 + cx)$

= $3ax^2 + 2bx + c$.

iii. Marginal revenue increases for, MR = $\frac{d}{dx}$(MR) > 0

$\Rightarrow$ 6ax + 2b > 0 $\Rightarrow$ x $\geq$ - $\frac{2b}{6a}$ $\Rightarrow$ x > -$\frac{b}{3a}$.

OR

Marginal revenue decreases for $\frac{d}{dx}$ (MR) $\leq$ 0

$\Rightarrow$ 6ax + 2b $\leq$ 0 $\Rightarrow$ x $\leq$ -$\frac{2b}{6a}$ $\Rightarrow$ x $\leq$ - $\frac{b}{3a}$.

37. i. Given P = ₹ 30,00,000, i = $\frac{7.5}{1200}$ =0.00625,

n = 12 $\times$ 20 = 240 months.

EMI = $\frac{3000000 \times 0.00625(1.00625)^{240}}{(1.00625)^{240}-1}$

$= \frac{3000000 \times 0.00625 \times 4.4608}{3.4608}$ = ₹ 24167.82.

ii. Given P = ₹ 30,00,000, i = $\frac{7.5}{1200}$ =0.00625,

n = 12 $\times$ 20 = 240 months.

Interest paid in 150th payment = $\frac{\text{EMI}\left[(1+i)^{240-150+1}-1\right]}{(1+i)^{240-150+1}} = \frac{24167.82\left[(1.00625)^{91}-1\right]}{(1.00625)^{91}}$

$= \frac{24167.82 \times 0.7629}{1.7629}$ = ₹ 10458.69

iii. Given P = ₹ 30,00,000, i = $\frac{7.5}{1200}$ =0.00625,

n = 12 $\times$ 20 = 240 months.

Principal paid in 150th payment = EMI - Interest paid in 150th payment

=₹ 24167.82 - ₹ 10458.69 = ₹ 13709.13.

OR

Given P = ₹ 30,00,000, i = $\frac{7.5}{1200}$ =0.00625,

n = 12 $\times$ 20 = 240 months.

Total interest paid = n $\times$ EM! - P = 240 $\times$ 24167.82 - 30,00,000 = ₹2800276.80

38. i. $\begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

ii. $\begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$

iii. $\begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$

OR

$\begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$